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Effects of Pollution Restrictions on Dynamic Investment Policy of a Firm[1]

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Abstract. The purpose of this paper is to determine the effects of different pollution standards on the firm's resource allocation decisions. To do so, a dynamic model of the firm is developed in which it is assumed that production causes pollution as an inevitable byproduct. Concerning its investment policy, we suppose that the firm can choose between investing in productive capital goods and investing in abatement efforts.

It is shown that, in some cases, future abatement expenses have a negative impact on the present level of productive investment, even if the pollution standard is not binding at the moment. This implies a really dynamic optimal investment policy for the firm, which cannot be obtained within a comparative static analysis.

Key Words. Optimal control, dynamics of the firm, investment policy, pollution standards.

1. Introduction

Many production processes damage the environment, and this is a subject of increasing concern in the world of today. An important question in this respect is what kind of policy instruments the government, in its role as social planner, should choose to reduce the level of pollution. In Ref. 1, it is found that pricing methods, like taxes and marketable permits, have important efficiency advantages over standards. The authors of Ref. 1

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derived that efficiency requires that abatement methods must be exploited such that marginal abatement costs are equal across all methods. In the case of standards, it is an impossible task for the government to fix all standards such that marginal abatement costs are equal, while by imposing a tax or creating a market for pollution permits, marginal abatement costs are automatically equalized, because all polluters will abate such that marginal abatement costs equal the tax or the price of the permit.

But, as argued in Ref. 2, in practice political and technological constraints can occur that lead to a poor performance of these pricing instruments. Therefore, it is important to recognize that the nature of individual environmental problems can affect dramatically the choice of preferred policy instruments. Thus for example, for highly localized pollution problems with threshold damage functions (e.g., Ref. 3, Fig. 8.3), source-specific standards may be particularly desirable. Moreover, in Ref. 4 it is obtained that, by analyzing a multiple country game, standards should be preferred to taxes, because the use of standards permits greater commitment by producers and this allows them to earn higher surpluses. Furthermore, marketable permits are not successful when the number of competitors is small (Ref. 5).

Taking all this into account, we conclude that the ideal policy package contains a mixture of instruments, with taxes, marketable permits, standards, and even moral persuasion, each used in certain circumstances to regulate the sources of environmental damage (cf. Ref. 6, p. 190). Therefore, from a management point of view, it is important to know how the firm must react to the imposition of each of these instruments. This paper focuses on the effects standards have on optimal dynamic firm behavior. The implications of an emission tax can be found in Ref. 7 and Ref. 8, while determining the optimal dynamic behavior of a firm that has to buy permits for polluting the environment is done in Ref. 9.

When economists refer to pollution standards, they almost universally mean uniform restrictions on pollution emissions. However, in practice, standards take many forms: not only emission restrictions, but also restrictions on pollution per unit output or per unit input, restrictions on the use of a polluting input, or mandated use of a particular pollution-control technology. In Ref. 10, the implications of a range of standards are studied within a comparative static framework. The purpose of this paper is to extend this work by establishing the effects of standards in a dynamic environment.

In Section 2, the model is formulated, while in Section 3 we examine the effects of introducing five different kinds of standards, namely: a fixed level of emission, a fixed level of emission per unit output, a fixed level of emission per unit input, a fixed level of output, and a fixed level of input. In Section 4, we compare the results of the different standards.

2. Model

Consider a firm that owns a stock of capital goods K. In order to concentrate on pollution effects, rather than capital–labor substitutions, we assume that the labor–capital ratio is fixed,

$$L = lK, \tag{1}$$

in which L is the stock of labor and l is the labor–capital ratio ($l > 0$ and constant).

The firm produces a homogeneous output, and production will be proportional to the input,

$$Q = qK = qL/l, \tag{2}$$

in which Q is the production rate and q is the capital productivity ($q > 0$ and constant).

Although precedents of this assumption of fixed proportions in the production function exists (for instance, Ref. 11), admittedly this is not a trivial assumption and is made for analytical simplicity. In the rest of the paper, we will mention it explicitly when results change significantly if this assumption is relaxed.

We assume that the sales level is an increasing function of production with decreasing marginal sales,

$$G(Q) = P(Q)Q, \tag{3}$$

in which $P = P(Q)$ is the selling price per unit of production and $G = G(Q)$ is the sales rate $[G \geq 0, G' > 0, G'' < 0, G(0) = 0]$. In particular, $G' > 0$ says that the demand function is elastic with respect to the price [i.e., $-(P/Q)(dQ/dP) > 1$], and $G'' < 0$ is equivalent to assuming[3] that $2P' < -P''Q$.

Due to the fixed labor–capital ratio, the earnings (the difference between sales and labor costs) are a concave function of K. By using (1)–(3), this can be expressed as follows:

$$S(K) = [qP(qK) - wl]K, \tag{4}$$

in which $S(K)$ is the earnings rate $[S \geq 0, S' > 0, S'' < 0, S(0) = 0]$ and w is the wage rate ($w > 0$ and constant).

The capital stock decreases by depreciation and can be increased by productive investment,

$$\dot{K} = I - aK, \qquad K(0) = K_0, \tag{5}$$

[3] The major conclusions of this paper are not affected if instead we assume that the firm faces a horizontal output demand curve, i.e., P is constant.

in which I is the rate of productive investment and a is the depreciation rate.

The firm also produces pollution. Following Ref. 3, pp. 152–154, the emission–output ratio can be reduced by investment in clean technology. In this way, the emission–output ratio, and thus the emission–capital ratio [cf. (2)], becomes a decreasing function of abatement investment,[4]

$$E = \alpha(A)Q = e(A)K, \tag{6}$$

in which E is the amount of emissions, A is the abatement investment rate, $\alpha(A)$ is the emission–output ratio ($\alpha' < 0$, $\alpha'' > 0$), and $e(A)$ is the emission–capital ratio [$e(A) = q\alpha(A)$, $e' < 0$, $e'' > 0$].

Note that $A = 0$ is associated with the production technology that would be chosen by a profit-maximizing firm in the absence of any environmental regulations. Hence, the emission–output ratio associated with this technology is $\alpha(0)$. The assumption of diminishing returns to abatement investments ($\alpha'' > 0$) seems to be realistic; see Ref. 12, Chapter 2 for practical examples.

Abatement investment is nonnegative,

$$A \geq 0. \tag{7}$$

The firm is assumed to behave so as to maximize the net cash flow stream. After supposing that, due to adjustment costs, convex costs are associated to productive investments, and abatement investment faces a horizontal supply curve, we arrive at the following objective function:

$$\max \int_0^\infty \exp(-rt)[S(K) - vA - C(I)]\, dt, \tag{8}$$

in which $C(I)$ are the costs of productive investment [$C(0) = 0$, $C' > 0$, $C'' > 0$], r is the discount rate, and v is the price of a unit of abatement investment ($v > 0$ and constant).

To facilitate the analysis, we introduce the following additional assumption concerning the shape of the emission–capital ratio function:

$$2(e')^2 < ee''. \tag{9}$$

3. Constraints

Here, we study the implications of five kinds of different pollution standards. We start by incorporating a maximal emission standard and

[4]Notice that emissions are a linear function of K, because Q is a linear function of K.

proceed by introducing restrictions on emission level per unit output, emission level per unit input, output level, and input level, respectively.

But, in order to serve as a benchmark, we first derive the solution in the case where there are no environmental regulations. Then, there is no incentive for abatement investment, implying that $A = 0$. Hence, the control problem left to be solved is given by (5) and (8) with $A = 0$. The current value Hamiltonian equals

$$H = S(K) - C(I) + \lambda(I - aK). \tag{10}$$

The necessary conditions are

$$\lambda = C'(I), \tag{11}$$

$$\dot{\lambda} = (r + a)\lambda - S'(K). \tag{12}$$

These conditions are also sufficient for optimality provided that the following transversality condition holds for every feasible solution $\tilde{K}$ (cf. Ref. 13):

$$\lim_{t \to \infty} \exp(-rt)\lambda(t)[\tilde{K}(t) - K(t)] \geq 0. \tag{13}$$

From (11) and (12), we obtain

$$\dot{I} = [1/C''(I)]\{(r + a)C'(I) - S'(K)\}. \tag{14}$$

The steady state follows from (5) and (14) and can be expressed as

$$\hat{I} = a\hat{K}, \tag{15}$$

$$S'(\hat{K}) = (r + a)C'(\hat{I}). \tag{16}$$

The determinant of the Jacobian of the system (5) and (14), which is evaluated in $(\hat{I}, \hat{K})$, is negative so that the dynamics corresponds to a saddle point.

After solving the differential equation (12), substituting (11) into this relation, and using (16) as a fixed point, we derive the following condition which holds for each t:

$$\int_t^\infty S'(K(s)) \exp(-(a + r)(s - t))\, ds - C'(I(t)) = 0, \tag{17}$$

where the left-hand side is the net present value of marginal investment. For an interpretation, consider the acquisition of an extra unit of capital at time t. The firm incurs an extra expense at time t in amount of C'. On the other hand, the marginal unit of capital generates, as of time t, a stream of earnings S'. This stream is corrected for depreciation by multiplication by $\exp[-a(s - t)]$ and is discounted to time t by multiplication by $\exp[-i(s - t)]$. Condition (17) then states that the net present value of marginal investment equals zero. Hence, the fundamental economic principle of balancing marginal earnings with marginal expenses applies; see Ref. 14.

3.1. Standard as a Set Level of Emission. Let Z_E be the numerical standard set when emissions are regulated by the amount of total pollution permissible per unit of time. From (6), we derive that the following constraint has to be imposed[5]:

$$e(A)K \leq Z_E. \tag{18}$$

Now, we need to solve the control problem represented by (5), (7), (8), and (18). We first note that A occurs only in the objective and in the constraints, but not in the system dynamics (5). Therefore, the problem can be solved by application of a two-step procedure; see, e.g., Ref. 13, pp. 397–402, and Ref. 16.

Step 1. For every fixed K, solve the static optimization problem

$$\max_A \{-vA \mid e(A)K \leq Z_E;\, A \geq 0\}. \tag{19}$$

After deriving the Kuhn–Tucker conditions, we obtain that the solution of (19) is given by $A = A_E(K)$, where

$$A_E(K) = \begin{Bmatrix} 0 \\ A_{E2}(K) \end{Bmatrix}, \qquad \text{for } K \begin{Bmatrix} \leq Z_E/e(0) \\ > Z_E/e(0) \end{Bmatrix}; \tag{20}$$

here, $A_{E2}(K)$ is an implicit function that satisfies

$$e(A_{E2}(K))K = Z_E. \tag{21}$$

The derivatives of this abatement function are given by

$$A'_{E2}(K) = -e/e'K > 0, \tag{22}$$

$$A''_{E2}(K) = e\{2(e')^2 - ee''\}/\{(e')^3K^2\} > 0. \tag{23}$$

The inequality in (23) is due to assumption (9). Hence, A_E is a convex nondecreasing function of K.

Step 2. Solve the control problem represented by (5) and (8) for $A = A_E(K)$. The Hamiltonian is given by

$$H_E = S(K) - vA_E(K) - C(I) + \lambda_E(I - aK). \tag{24}$$

[5]If emissions are not linearly dependent on K, we get a far more complicated (stability) analysis with possible occurrence of history-dependent equilibria (see Ref. 15).

The necessary conditions are

$$\lambda_E = C'(I), \tag{25}$$

$$\dot{\lambda}_E = (r + a)\lambda_E - S'(K) + vA'_E(K). \tag{26}$$

Because $A_E(K)$ is convex, these conditions are also sufficient for optimality provided that a transversality condition like (13) is satisfied. Due to (25) and (26), we get

$$\dot{I} = [1/C''(I)]\{(r + a)C'(I) - S'(K) + vA'_E(K)\}. \tag{27}$$

Now, we carry out a state-control phase plane analysis for the differential equation system (5), (27). Due to (20), (22), and (27), we derive that the $\dot{I} = 0$ isocline jumps downward when K equals $Z_E/e(0)$. From Fig. 1, we obtain that there are three possible configurations.[6]

Because of (20), the $\dot{I} = 0$ isocline, which in Fig. 1 is denoted by $\dot{I}_E = 0$, consists of two parts. For $K < Z_E/e(0)$, it holds that $A_E(K) = 0$ [cf. (20)]; therefore, the $\dot{I}_E = 0$ isocline coincides with the $\dot{I} = 0$ isocline of the unregulated case [cf. (14), (27)]. Hence, if the equilibrium point is such that $\hat{K} < Z_E/e(0)$, then we have the same equilibrium point for the regulated

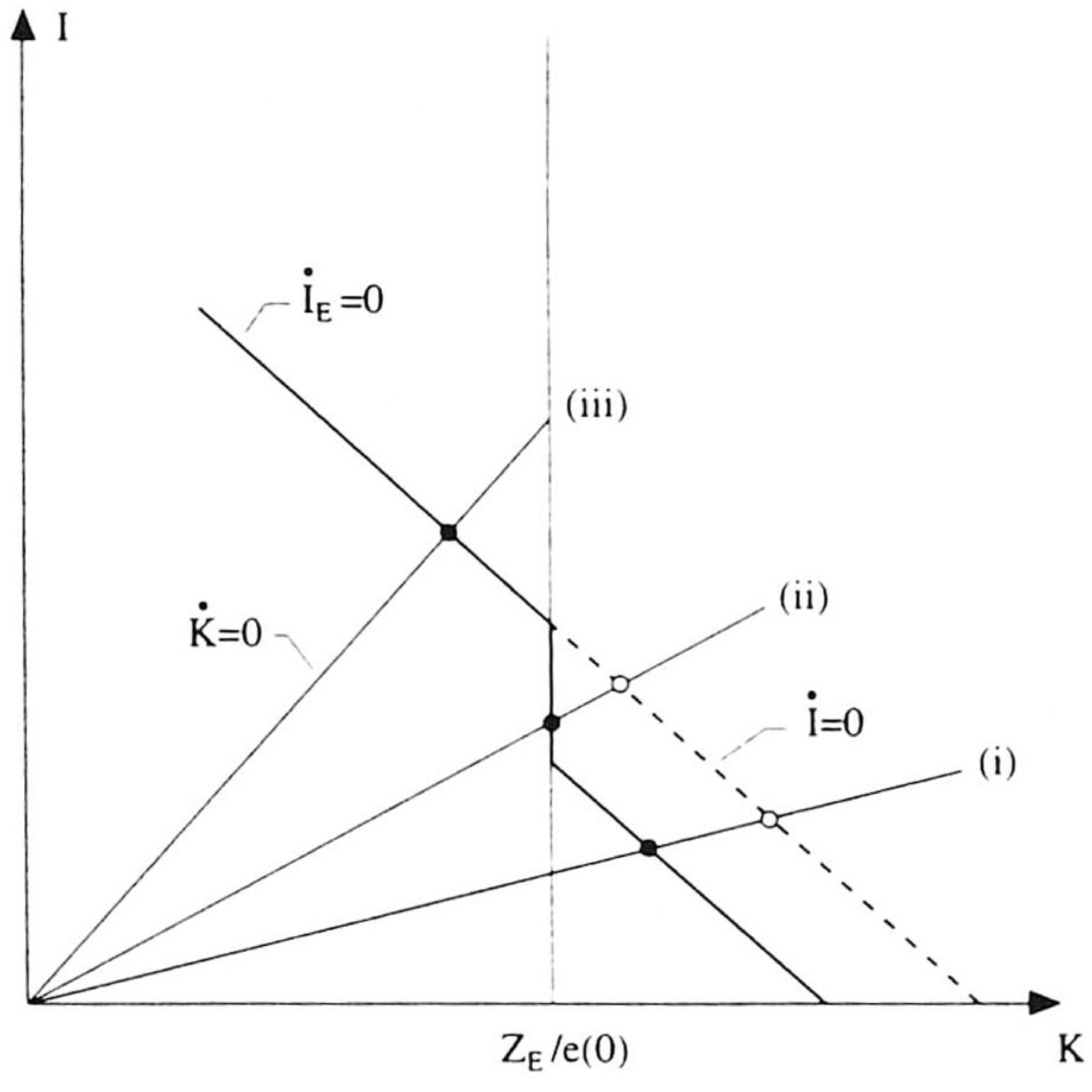
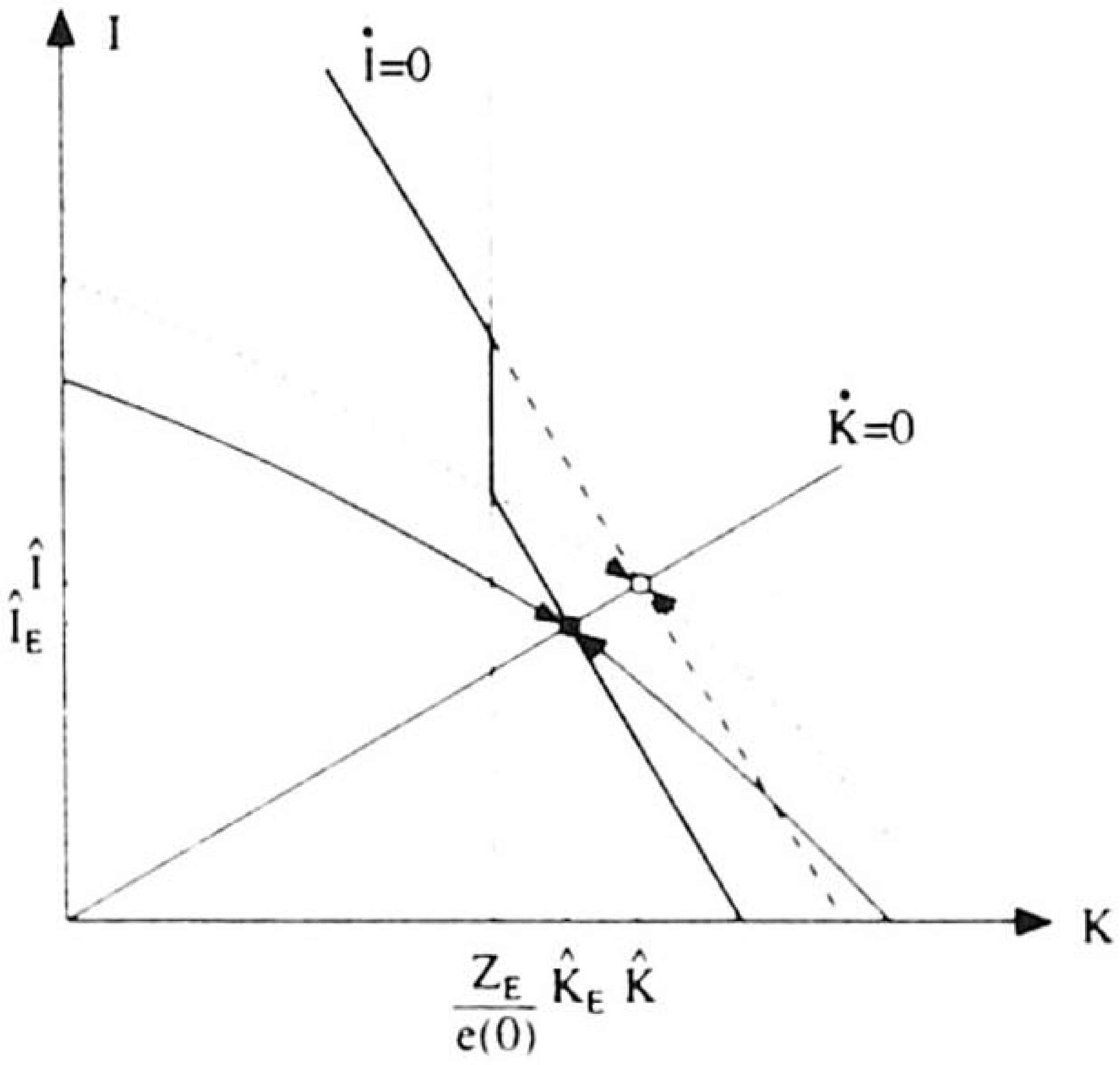
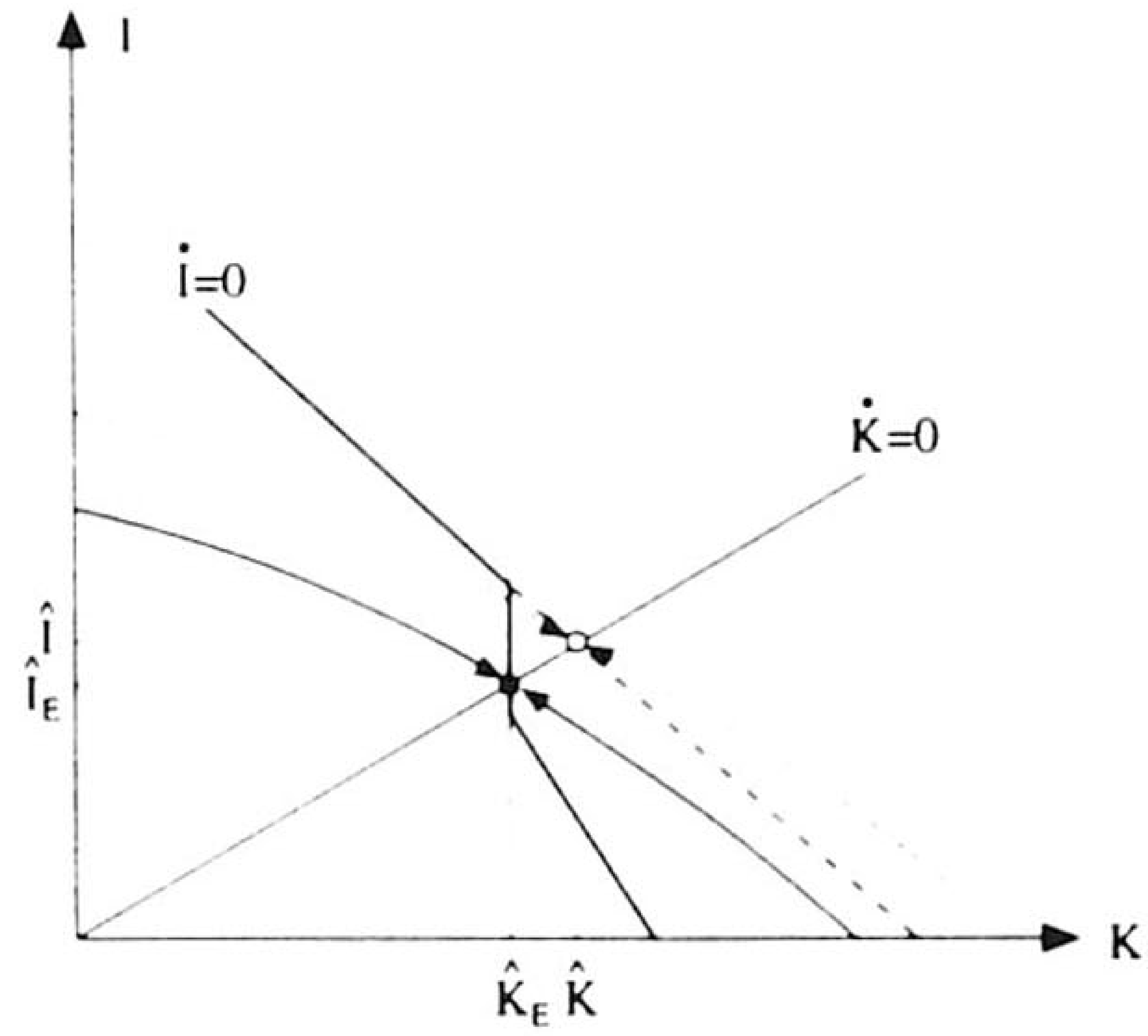


Fig. 1. Three possible configurations in the state-control phase diagram.

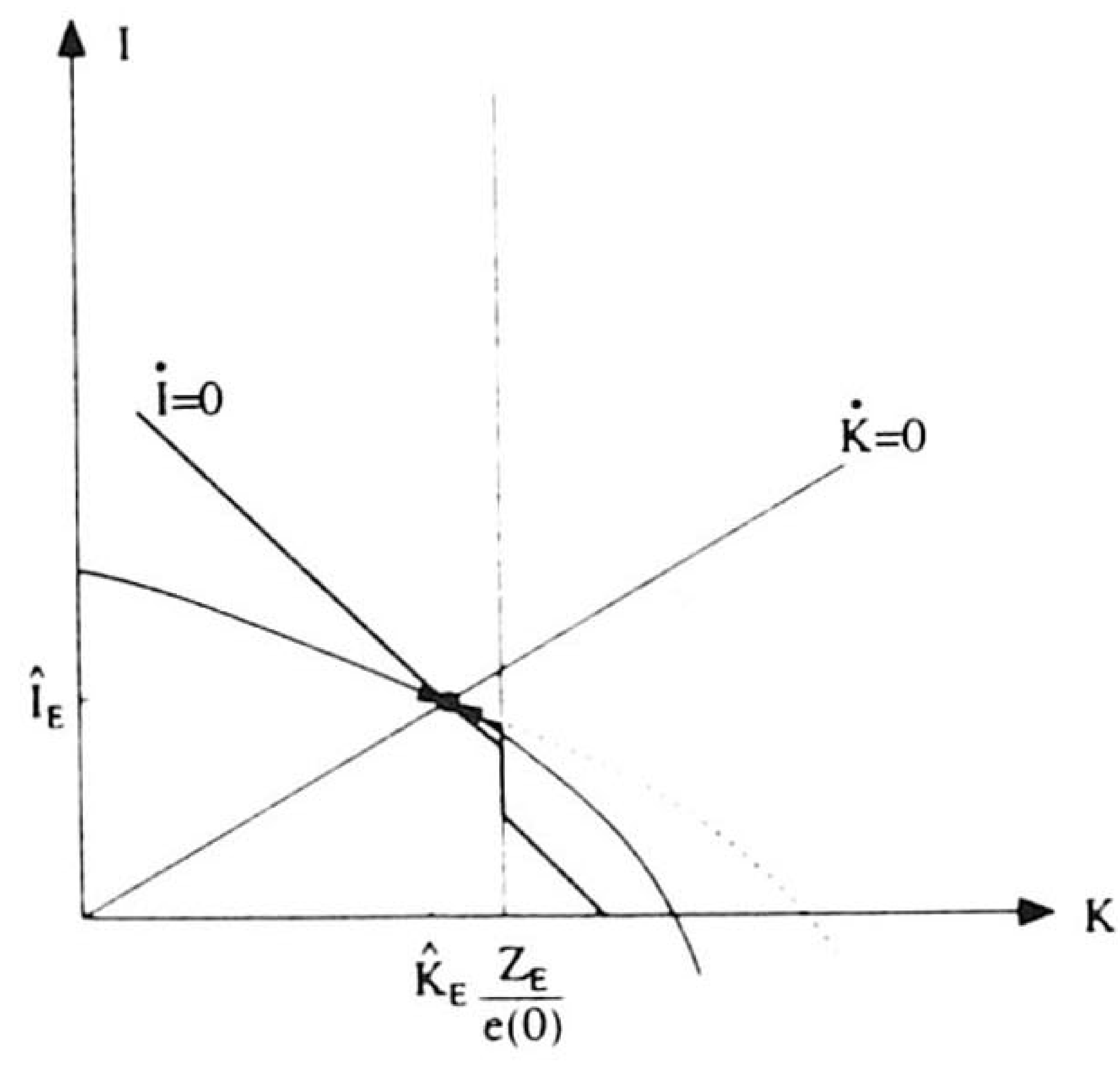
[6]In Figs. 1 and 2, the $\dot{I} = 0$ isoclines are drawn as straight lines. This is only for simplicity. The real shape depends on S'', C'', and A''.



(a)



(b)



(c)

Fig. 2. Optimal solutions in cases (i)–(iii): unregulated case (----); regulated case (

and the unregulated case; see configuration (iii) in Fig. 1. For $K > Z_E/e(0)$, the $\dot{I}_t = 0$ isocline of the regulated case is situated below the $\dot{I} = 0$ isocline of the unregulated case. Therefore, the equilibrium points differ [configurations (i), (ii)]; in Fig. 1, the equilibrium point of the unregulated case is denoted by a hollow circle and the equilibrium point of the regulated case by a full circle.

In Fig. 2, the three configurations are drawn in separate phase diagrams. Also here the saddle point of the unregulated case $(\hat{K}, \hat{I})$ is denoted by a hollow circle and of the regulated case $(\hat{K}_E, \hat{I}_E)$ by a full circle. In Fig. 2a, the equilibrium level of capital stock in the regulated case satisfies

$$S'(\hat{K}_E) = (r + a)C'(a\hat{K}_E) + vA'_E(\hat{K}_E). \tag{28}$$

Equation (28) says that, in equilibrium, marginal earnings equal the sum of marginal investment costs and marginal abatement expenses necessary to keep pollution equal to the standard level when capital stock increases marginally. It is easy to check that the determinant of the Jacobian is negative, so that this equilibrium point is a saddle point as well. Compared to the equilibrium level in the unregulated case [cf. (15), (16)], here the capital stock is lower, which is due to the abatement expenses that are forced by the standard.

In Fig. 2b, the equilibrium level of capital stock equals $Z_E/e(0)$ and satisfies

$$(r + a)C'(a\hat{K}_E) < S'(\hat{K}_E) < (r + a)C'(a\hat{K}_E) + vA'_E(\hat{K}_E). \tag{29}$$

From the first inequality in (29), we infer that marginal earnings exceed marginal investment costs, so it would be optimal for the firm to grow further when no abatement investments are necessary. But when the firm grows beyond $Z_E/e(0)$, then abatement expenditures are needed to meet the standard [cf. (20)]. Hence, marginal costs increase with the abatement costs, and from the second inequality in (29), we obtain that this implies that total marginal costs exceed marginal earnings. This means that it is optimal for the firm to keep the level of capital stock equal to $Z_E/e(0)$. From the first inequality in (29), we also derive that the equilibrium level of capital stock in the unregulated case exceeds the one in the regulated case.

In Fig. 2c, the equilibrium point is given by

$$S'(\hat{K}_E) = (r + a)C'(a\hat{K}_E). \tag{30}$$

Here, the equilibrium level is such that $\hat{K}_E < Z_E/e(0)$; hence, no abatement expenditures are necessary to meet the standard. Therefore, no abatement costs are contained in the marginal earnings–cost relation (30). Comparing (16) and (30) shows that the equilibrium levels of the regulated and the unregulated case are the same.

We see that imposing the emission standard only reduces the equilibrium level when $\hat{K} > Z_E/e(0)$. This brings us to the conclusion that large firms are influenced by an emission standard. This feature was also found in Ref. 10.

In Fig. 2a, the investment level is always lower in the regulated case, while in Fig. 2c this only holds for large values of capital stock, namely, for those values where abatement investments are needed to satisfy the standard. This corresponds to a firm that initially has grown large, but now has to contract, for instance, because of the fact that the output market has declined. In Fig. 2a, it turns out that investments are lower even when abatement expenditures are not yet required. This behavior can be confirmed by a net present value rule. Suppose that the firm starts out with a capital stock lower than $Z_E/e(0)$ and that this level is reached at time t_E. Then, from the optimality conditions, we can derive the following expression for the net present value of marginal investment (NPVMI)[7]:

$$\int_t^{\infty} S'(K(s)) \exp(-(a+r)(s-t))\, ds$$

$$-\int_{t_E}^{\infty} vA'_E(K(s)) \exp(-(a+r)(s-t))\, ds - C'(I(t)) = 0. \qquad (31)$$

Because the second term is negative, the investment level must be reduced compared to the unregulated case [cf. (17)] to keep the NPVMI equal to zero. This equation confirms that, at each moment of time, the firm reckons with future abatement expenditures when it determines its investment rate. The reason for this is that, when the firm invests one dollar at time t, capital stock increases with $\exp[-a(s-t)]$ at each time $s > t$, implying that at each time $s > t_E$ additional abatement expenditures are necessary to keep pollution equal to Z_E.

When we consider a firm starting out with a capital stock below $Z_E/e(0)$ in Fig. 2c, then K will remain below $Z_E/e(0)$. This implies that no future abatement expenditures are needed, so the second term in (31) disappears, which means that the NPVMI expression becomes equal to the

[7]Of course, if abatement expenditures take place only in the beginning, say within the interval $[0, t'_E]$, then the lower bound and upper bound of the integral in the second term of (31) would be 0 and t'_E, respectively.

one in the unregulated case [cf. (17)]. Therefore, the investment levels of the regulated and unregulated case coincide here.

3.2. Standard as Emission per Unit Output. Let Z_{EQ} be the standard expressed as a set level of pollution per unit output. According to (6), this amounts to

$$e(A)K \leq Z_{EQ}Q. \tag{32}$$

From (2), we obtain that this constraint can be rewritten as

$$e(A) \leq qZ_{EQ}. \tag{33}$$

Abatement expenditures are costly, and therefore it is optimal for the firm to put them as low as possible. In this way, optimal abatement investments are given by

$$A_{EQ} = \begin{cases} 0, & e(0) \leq qZ_{EQ}, \\ A_{EQ2}, & e(0) > qZ_{EQ}, \end{cases} \tag{34}$$

where A_{EQ2} satisfies

$$e(A_{EQ2}) = qZ_{EQ}. \tag{35}$$

Hence, abatement expenditures are constant over time and also independent of the stock of capital goods.[8] Abatement expenditures are not needed when the emission–output ratio without abatement effort $e(0)/q$ already satisfies the standard. But the standard can also be so restrictive that, over the whole planning period, the firm must assign a constant amount of money to abatement investments in order to reduce the emission–output ratio. As said before, A does not depend on K and thus the remaining control problem can be solved independently of A. Consequently, this leads to the same productive investment policy as in the unrestricted case, and thus (15)–(17) also apply here.

From an economic point of view, the level of productive investment, being unaffected by the emission per unit output standard, can be explained by noting that here having an increased stock of capital goods does not

[8] Of course, this outcome is due to the linear dependence of pollution and production on capital stock. If for instance, emissions are represented by $e(A, K)$, then this standard, being binding, leads to

$$(\partial e/\partial A)\, dA = (qZ_{EQ} - \partial e/\partial K)\, dK.$$

Hence, abatement investments are not constant, but they depend on the development of K.

have consequences for future abatement expenses. Therefore, abatement costs do not occur in the NPVMI relation [cf. (17)]. Because abatement expenses are the same regardless of the size of the firm at any time, an emission per unit output standard seems to be unfavorable for the firm when it is small.

3.3. Standard as Emission per Unit Input. Two cases are possible here: regulating pollution per unit of capital goods or regulating pollution per unit of abatement investments. In the USA, the Environmental Protection Agency preferred a pollution per unit of abatement investments standard out of concern that plants would meet the standard solely by dilution and not by cleanup; see Ref. 10, p. 626. The first possibility leads to the following mathematical representation:

$$e(A)K \leq Z_{EK}K. \tag{36}$$

After dividing both sides by K, we conclude that imposing this constraint is similar to imposing the emission per unit output standard; therefore, the results stated in the previous subsection also apply here.

Regulating pollution per unit of abatement investments leads to the following constraint:

$$e(A)K \leq Z_{EA}A. \tag{37}$$

Since abatement investments are costly, there is no reason for the firm to invest in abatement activities more than required by the standard. Hence, throughout the whole planning period, A will be an implicit function of K, $A = A_{EA}(K)$, that satisfies

$$e(A_{EA}(K))K = Z_{EA}A_{EA}(K). \tag{38}$$

From (38), we obtain

$$A'_{EA}(K) = e/(Z_{EA} - e'K) > 0, \tag{39}$$

$$A''_{EA}(K) = \{2e'Z_{EA} + K(-2(e')^2 + ee'')\}e/(Z_{EA} - e'K)^3. \tag{40}$$

Positivity of the first derivative follows from the negative sign of e'. The sign of the second derivative is ambiguous, because in the numerator the first term is negative, while the second term is positive due to assumption (9).

As Step 2, we now need to solve the control problem represented by (5) and (8) for $A = A_{EA}(K)$. The Hamiltonian is given by

$$H_{EA} = S(K) - vA_{EA}(K) - C(I) + \lambda_{EA}(I - aK). \tag{41}$$

Then, the necessary conditions are

$$\lambda_{EA} = C'(I), \tag{42}$$

$$\dot{\lambda}_{EA} = (r + a)\lambda_{EA} - S'(K) + vA'_{EA}(K). \tag{43}$$

From these conditions, we obtain

$$\dot{I} = [1/C''(I)]\{(r + a)C'(I) - S'(K) + vA'_{EA}(K)\}. \tag{44}$$

The steady-state value of capital stock can be obtained from (5) and (44),

$$S'(\hat{K}_{EA}) = (r + a)C'(a\hat{K}_{EA}) + vA'_{EA}(\hat{K}_{EA}). \tag{45}$$

Comparing (15)–(16) and (45) leads to the conclusion that this steady-state value is lower than in the unregulated case. The determinant of the Jacobian evaluated at the steady state equals $-a(r + a)C'' + S'' - vA''$. Notice that negativity of this determinant is not assured, because one of the two terms in the numerator of A'' is negative. However, it is likely that negativity of this term is compensated by the other term in the numerator of A'' and by the first two terms of the determinant. Therefore, here we suppose that the dynamics corresponds to a saddle point.

From the optimality conditions, we obtain the following NPVMI relation:

$$\int_t^T \{S'(K(s)) - vA'_{EA}(K(s))\} \exp(-(r + a)(s - t))\, ds - C'(I(t)) = 0. \tag{46}$$

Due to the fact that the firm abates such that the standard is just satisfied [cf. (38)], an additional investment expenditure immediately requires extra abatement expenses. Therefore, these are subtracted from marginal earnings in the NPVMI relation, which implies that the level of productive investment is lower than in the unregulated case.

The difference with the emission standard is that here, throughout the whole planning period, abatement expenses are required to meet the emission per unit abatement standard, while with the emission standard, this is only the case when capital goods exceed a certain level. Hence, unlike the emission standard, the emission per unit abatement standard does not favor the small firms.

3.4. Standard as a Set Level of Total Output. The constraint to be added is now

$$qK \leq Z_Q. \tag{47}$$

Because pollution (or pollution per unit output or input) is not directly restricted here, abatement expenses are not of any use to the firm, so throughout the whole planning period it holds that $A = 0$. Thus, the control problem to be solved is represented by (5), (8), with $A = 0$, and (47), where the latter is a pure state constraint. Of course, it has to be imposed that $K(0) \leq Z_Q/q$. By using the direct method (see Ref. 13), the Lagrangian equals

$$L = S(K) - C(I) + \lambda_Q(I - aK) + \mu_Q(Z_Q/q - K). \tag{48}$$

The necessary conditions are

$$\lambda_Q = C'(I), \tag{49}$$

$$\dot{\lambda}_Q = (r + a)\lambda - S'(K) + \mu_Q, \tag{50}$$

$$\mu_Q(Z_Q/q - K) = 0, \qquad \mu_Q \geq 0. \tag{51}$$

Due to the fact that the Hamiltonian is strictly concave in I, it is regular. Now, from Corollaries 6.2, 6.3a of Ref. 13, and due to satisfaction of constraint qualification (6.17) of Ref. 13, we get that I and λ_Q are continuous.

To obtain the optimal investment policy, we follow the approach of Ref. 13, pp. 218–219. First, notice that there are two possibilities.

(a) $K(t) < Z_Q/q$, $\forall t$. This implies that $\mu_Q = 0$, so that the optimality conditions are the same as in the unregulated case. Consequently, the optimal investment policy is here also given by (17).

(b) K reaches its upper bound at a certain point of time. Starting at $K = K_0$, we have to find a trajectory that satisfies the constraint as well as the necessary conditions everywhere. To do so, we intersect $K = Z_Q/q$ with the $\dot{K} = 0$ isocline and study the trajectory that ends in this intersection point, which is denoted by (K_Q, I_Q); see Fig. 3.

If we choose for $K = K_0$ the corresponding investment rate on this trajectory, then the point (K_Q, I_Q) is reached at a finite point of time t_Q. Then, it makes sense to choose the control $I = I_Q$ for $t \in (t_Q, \infty)$.[9]

The necessary conditions are of course satisfied for $t \leq t_Q$. For $t > t_Q$, it holds that $\lambda_Q = C'(I_Q)$. Hence, $\dot{\lambda}_Q = 0$, and we obtain, after noticing that $K_Q < \hat{K}$ and $\hat{K}$ satisfies [cf. (16)] $S'(\hat{K}) = (r + a)C'(\hat{I})$,

$$\mu_Q = S'(K_Q) - (r + a)C'(I_Q) > 0. \tag{52}$$

Hence, also for $t > t_Q$, the necessary conditions are fulfilled. Because K and λ are always finite on this trajectory, the sufficient conditions from

[9]Notice that $I > I_Q$ is not allowed, because then K would grow beyond K_Q implying that the maximal output standard is violated.

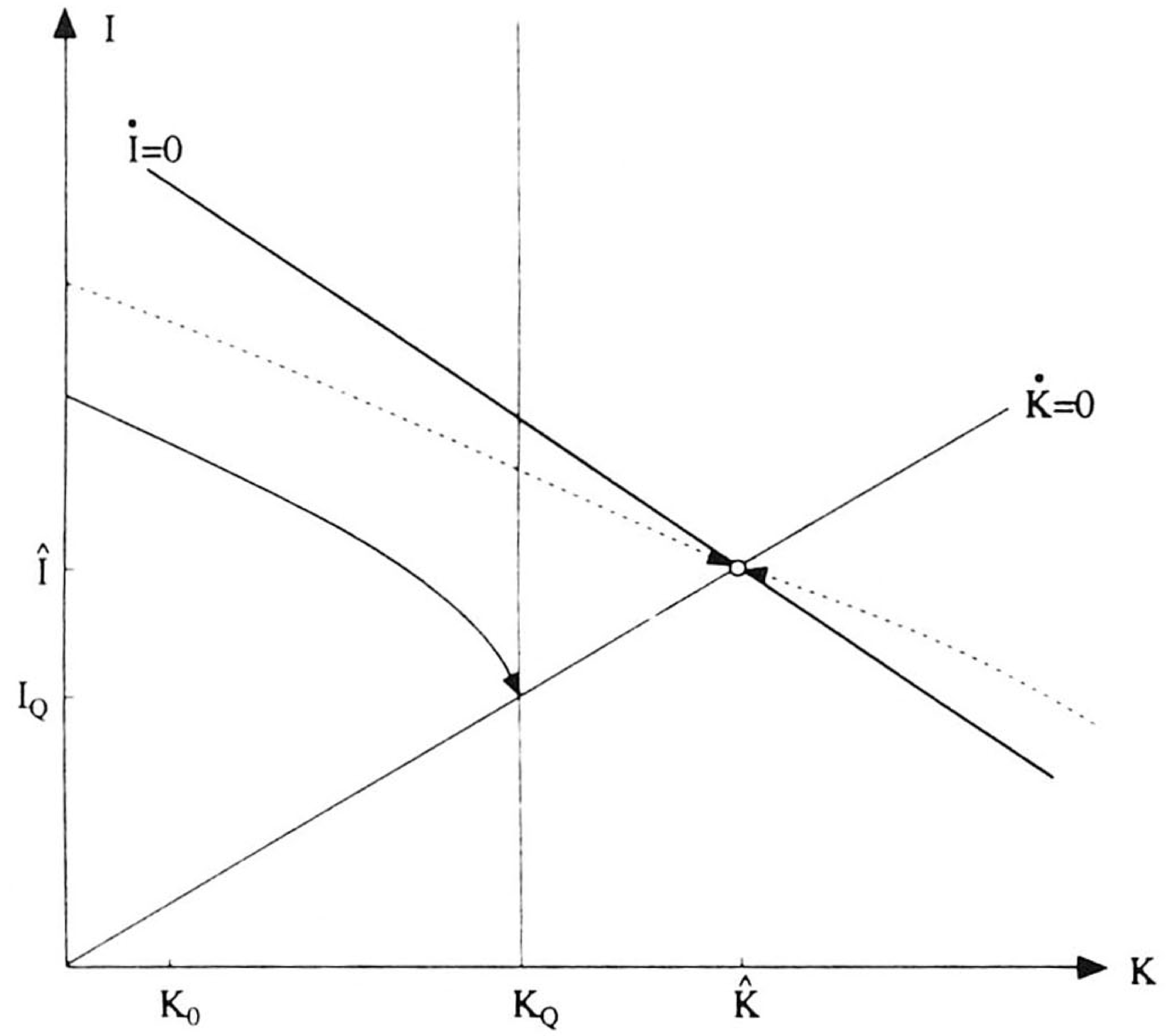


Fig. 3. Optimal solutions of the problem with maximal output standard (——) and the problem with no constraints (----).

Theorem 7.5 in Ref. 13 are satisfied, which means that the constructed solution is really optimal.

From the figure, we infer that, compared to the unrestricted problem, the investment rate is lower. Hence, as in the problem with the emission standard, in determining its investment policy the firm seems to reckon with the fact that the maximal output level will be reached after a while. This is confirmed by the NPVMI relation, which has the following form:

$$\int_t^\infty S'(K(s)) \exp(-(a+r)(s-t))\, ds$$
$$-\int_{t_Q}^\infty \mu_Q(s) \exp(-(a+r)(s-t))\, ds - C'(I(t)) = 0. \tag{53}$$

So, as in the previous solutions, the firm's investment policy has a really dynamic structure. Therefore, such a result cannot be obtained within a comparative static context (cf. Ref. 10). One of the results that coincides with this work is that the use of both inputs (K as well as A) decreases in this case.

3.5. Standard as a Set Amount of Input. This standard takes two forms. A maximum can be set on the stock of capital goods; alternatively, imposing a minimum level on the use of an abatement input captures the effect of imposing a particular pollution-control technology on a firm. The first possibility leads to the following mathematical representation:

$$K \leq Z_K. \tag{54}$$

After comparing (47) and (54), we conclude that setting a maximum on the capital stock will have the same effect as restricting the level of total output; therefore, the results stated in the previous subsection also apply here.

Imposing a minimum level on the level of abatement investment gives the following constraint:

$$A \geq Z_A. \tag{55}$$

The optimal control problem to be solved consists of the expressions (5), (8), and (55). Since abatement investments have to be paid for, the optimal policy is to put them as low as possible; i.e., $A(t) = Z_A$ for all t. Furthermore, the firm will apply the same productive investment policy as in the unregulated case, because abatement costs will not be influenced by an increase of capital goods. Hence, the level of productive investment satisfies the NPVMI relation (17).

4. Comparisons of Different Standards

As in Ref. 10, comparisons among the standards can be made only when they are normalized. Here, we normalize the standards such that, in each steady state, the firm produces the same amount of pollution. Also, we consider only those solutions where the standard has the biggest impact. For example, in case of the emission standard E, we arrived at three solutions; here, we pick that solution where abatement expenses are needed at the end to satisfy the standard.

As was noted in the previous section, restricting the emission per unit output (EQ) and restricting the emission per unit capital good (EK) lead to the same outcome, and this also holds for the maximal output standard (Q) and the maximal capital stock standard (K). Further, we derived that abatement expenditures are constant when the emission per unit output standard and the minimal abatement standard (A) are imposed. Because the productive investment policy coincides under these two standards and normalization requires that the amount of pollution be the same in the steady state, the abatement expenditures must be the same too; thus, the performance of standards EQ and A will be equal after normalization. This brings us to the conclusion that, after normalizing the standards, we have to consider four different solutions, namely the solutions resulting from imposing the emission standard, the emission per unit output standard, the emission per unit abatement investment standard, and the maximal output standard, respectively. Together with the unregulated case, they are depicted in Fig. 4, in which it is assumed that the firm starts out with a rather low level of capial stock.

Because the productive investment policy is the same, the development of capital stock over time in the unrestricted case coincides with the solution where the emission per unit output is restricted. We see that, over the whole planning period, this emission per unit output standard gives the highest level of capital stock. Because the firm does not spend money on abatement efforts in the case of a maximal output standard and the amount of emission must be equal in the end, the level of capital stock will be mostly reduced in this case. That the steady-state capital stock in the case of an emission per unit of abatement investment standard exceeds the one under an emission standard can be derived from (28), (45) and the fact that $A'_E(K) > A'_{EA}(K)$; cf. (22), (39).

As just mentioned, restricting emission per unit output gives the highest level of capital stock. Then, abatement expenditures must also be at the highest level, because emissions are the same in the end. In the case of an emission standard, we first have a period of zero abatement investment and it becomes positive as soon as the standard level of emission is reached. Contrary to this, in the case of the emission per unit abatement investment standard, the firm will carry out abatement investments during the whole planning period.

Of course, the amount of emission reaches the highest level in the unrestricted case. For the different standards, it holds that the amount of emission is the same in the end, due to the normalization. Unlike the other standards, this emission level is already reached within a finite time period when an emission standard is imposed. At the start of the planning period, emissions are mostly reduced in the case of an emission per unit output standard, because imposing the latter results in a high constant level of abatement expenditures over the whole planning period.

Next, consider the practically interesting case where the firm finds itself at the unregulated steady-state capital stock level when pollution regulation suddenly goes into effect.[10] Then, the firm immediately has to adjust its capital stock and/or abatement effort in order to meet the specific

[10]Notice that it is probably more realistic to give the firm some time to fulfill the requirements of the standard. In Ref. 17, the case is analyzed where the firm anticipates beforehand on the moment that regulation comes into force; see also Ref. 18.

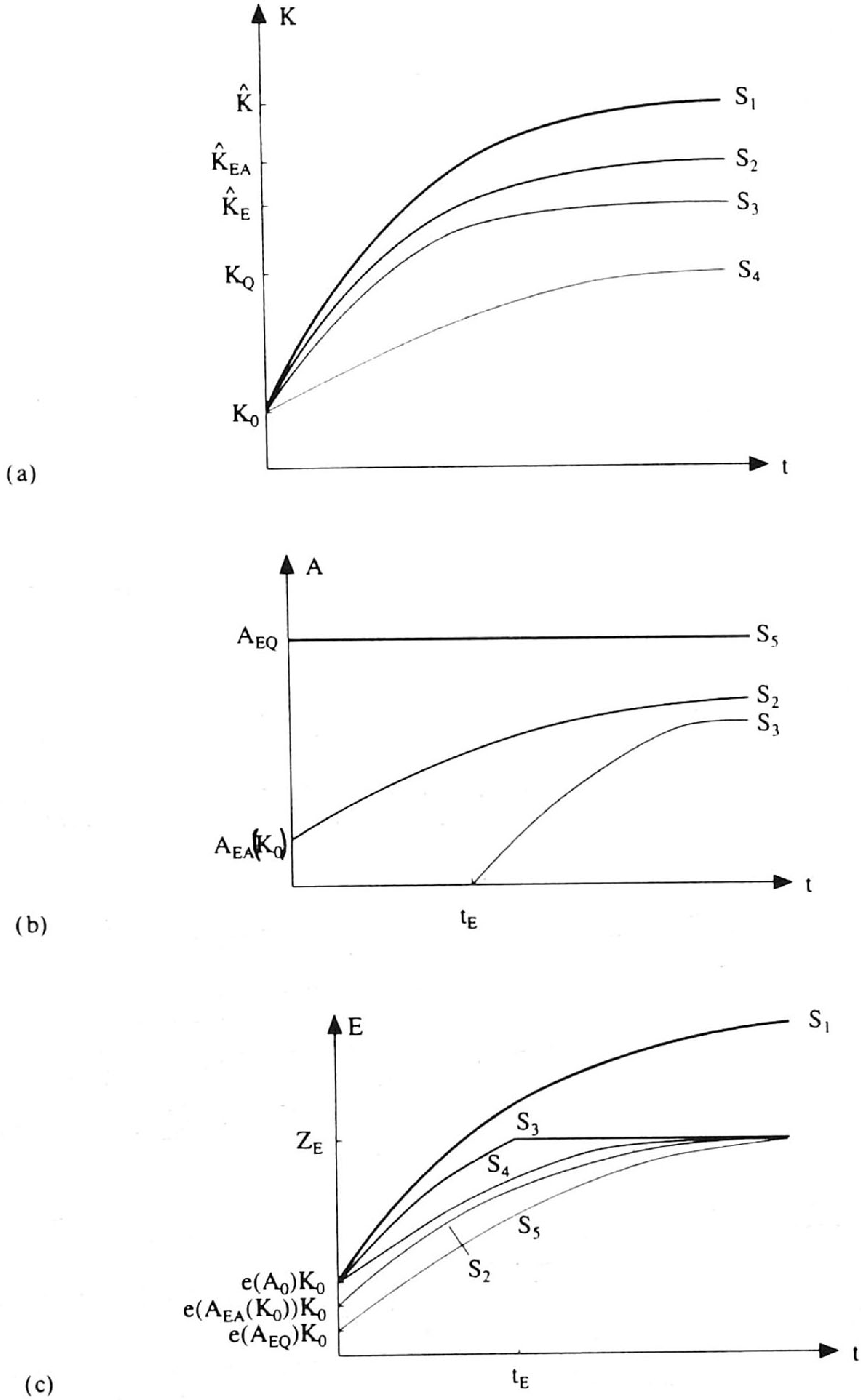


Fig. 4. Capital stock, abatement investment, and emission amount as functions of time for five different solutions where $K_0 < \hat{K}$: unregulated solution (S_1), emission standard solution (S_3), emission per output standard solution (S_5), emission per abatement investment standard solution (S_2), and maximal output standard solution (S_4).

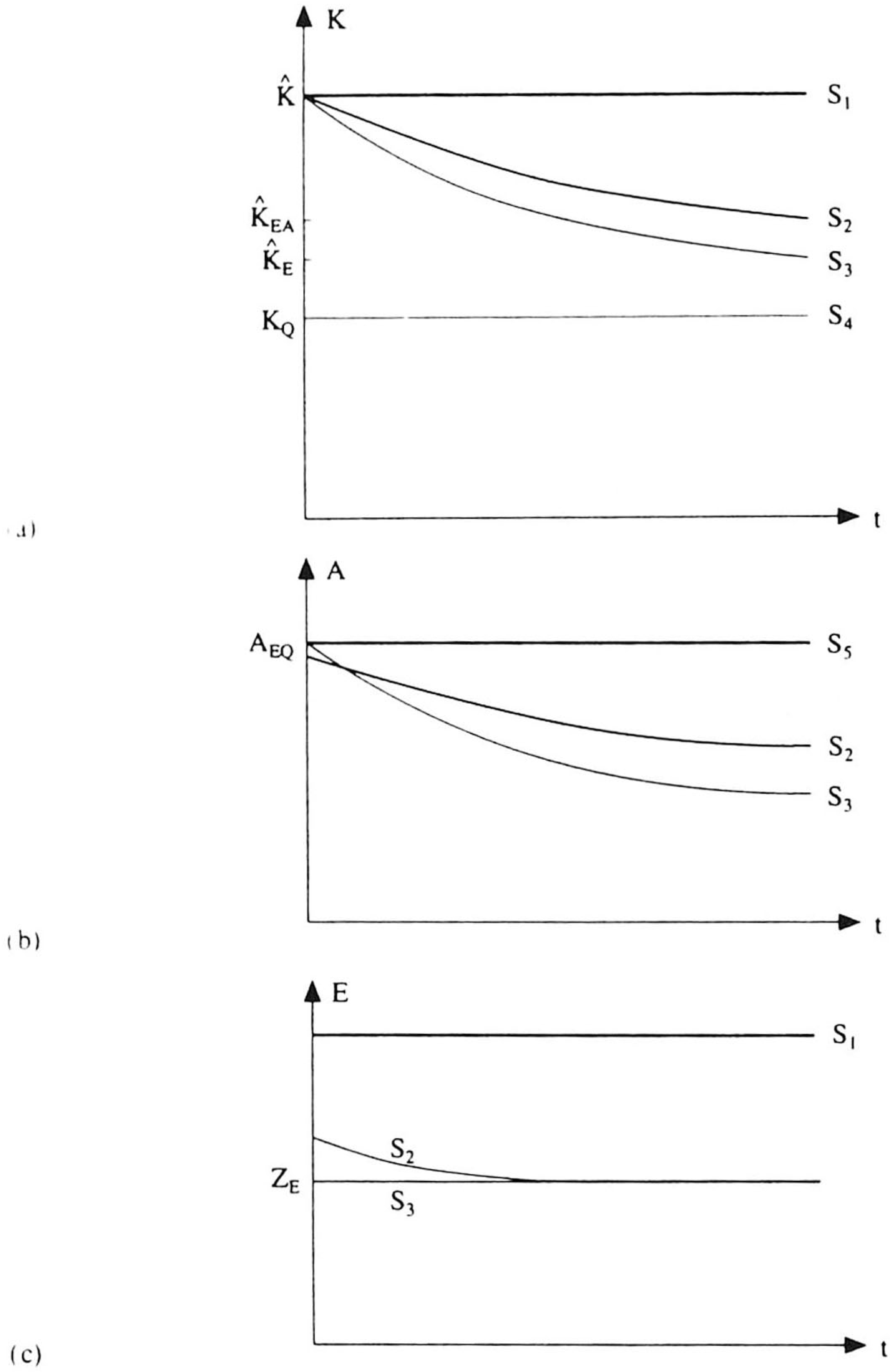


Fig. 5. Capital stock, abatement investment, and emission amount as functions of time for five different solutions where $K_0 = \hat{K}$: unregulated solution (S_1), emission standard solution (S_3), emission per output standard solution (S_5), emission per abatement investment standard solution (S_2), and maximal output standard solution (S_4).

standard. The optimal way of doing so under different standards is shown in Fig. 5.

The two extreme ways in which emission can be reduced occur under the maximal output standard and the emission per unit output standard. In the first case, the firm immediately reduces emissions only by a downward jump of capital stock, thus decreasing production; in the latter case, capital stock remains at the same level and emissions are solely reduced by carrying out a large amount of abatement investments. Intermediate solutions, thus at the same time investing in abatement input and reducing production, occur when an upper bound on emission or on emission per unit abatement investment is imposed.

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